

# Constraints on $R^n$ gravity from precession of orbits of S2-like stars

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We study some possible observational signatures of  $R^n$  gravity at Galactic scales and how these signatures could be used for constraining this type of  $f(R)$  gravity. For that purpose, we performed two-body simulations in  $R^n$  gravity potential and analyzed the obtained trajectories of S2-like stars around Galactic center, as well as resulting parameter space of  $R^n$  gravity potential. Here, we discuss the constraints on the  $R^n$  gravity which can be obtained from the observations of orbits of S2-like stars with the present and next generations of large telescopes. We make comparison between the theoretical results and observations. Our results show that the most probable value for the parameter  $r_c$  in  $R^n$  gravity potential in the case of S2-like stars is  $\sim 100$  AU, while the universal parameter  $\beta$  is close to 0.01. Also, the  $R^n$  gravity potential induces the precession of S2-like stars orbit in opposite direction with respect to General Relativity, therefore, such a behavior of orbits qualitatively is similar to a behavior of Newtonian orbits with a bulk distribution of matter (including a stellar cluster and dark matter distributions).

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## I. INTRODUCTION

Power-law fourth-order theories of gravity have been proposed like alternative approaches to Newtonian gravity [1, 2]. In this paper we study possible application of  $R^n$  gravity on Galactic scales, for explaining observed precession of orbits of S-stars, as well as whether these observations could be used for constraining this type of  $f(R)$  gravity [3].

S-stars are the bright stars which move around the massive black hole in the center of our Galaxy [4–6]. For one of them, called S2, there are some observational indications that this orbit deviates from the Keplerian case due to relativistic precession [4]. Besides, an extended dark mass which probably exists in the Galactic center, could also contribute to pericenter precession of the S2, but in the opposite direction [4, 7]. Progress in monitoring bright stars near the Galactic Center have been made recently [4]. With the Keck 10 m telescope, the several stars orbiting the black hole in Galactic Center have been monitored, and in some cases almost entire orbits, as, for example, that of the S2 star, have been observed, allowing an unprecedented description of the Galactic Center region [4]. The astrometric limit for S2 star orbit is today around 10 mas and within that limit one can not say for sure that S2 star orbit really deviates from the Newtonian case. In the future, it will be possible to measure the positions of some stars with astrometric errors several times smaller than errors of current observations and

that is why we will consider here even smaller astrometric limits.

Capozziello et al. [2] investigated the possibility that the observed flatness of the rotation curves of spiral galaxies is not evidence for the existence of dark matter (DM) haloes, but rather a signal of the breakdown of General Relativity (GR). They found a very good agreement between the theoretical rotation curves and the data using only stellar disc and interstellar gas when the slope  $n$  of the gravity Lagrangian is set to the value  $n = 3.5$  (giving  $\beta = 0.817$ ), obtained by fitting the Type Ia supernova Hubble diagram with the assumed power-law  $f(R)$  model and without dark matter [2].

Frigerio Martins and Salucci [8] have also investigated the possibility of fitting the rotation curves of spiral galaxies with the power-law fourth-order theory of gravity, without the need for dark matter. They show that, in general, the power law  $f(R)$  version could fit the observations well, with reasonable values for the mass model.

Recently, gravitational microlensing has been investigated in the framework of the weak field limit of fourth order gravity theory [9]. The solar system data (i.e. planetary periods) and light bending due to microlensing can be used to put strong constraints on the parameters of this class of gravity theories. In paper [9] it was found that these parameters must be very close to those corresponding to the Newtonian limit of the theory. In paper [10] the authors discuss the constraints that can be obtained from the orbit analysis of stars (as S2 and S16) moving inside the DM concentration. In particular, consideration of the S2 star apoastron shift may allow improving limits on the DM mass and size.

Rubilar and Eckart [11] investigated the properties of

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stellar orbits close to central mass and the corresponding connection with current and (near) future observational capabilities. They showed that the orbital precession can occur due to relativistic effects, resulting in a prograde shift, and due to a possible extended mass distribution, producing a retrograde shift. Both, prograde relativistic and retrograde Newtonian periastron shifts will result in rosette shaped orbits. Weinberg et al. [12] discuss physical experiments achievable via the monitoring of stellar dynamics near the massive black hole at the Galactic Center with a diffraction-limited, next-generation, extremely large telescope (ELT). They demonstrate that the lowest order relativistic effects, such as the prograde precession, will be detectable if the astrometric precision become less than 0.5 mas.

In this paper we continue to investigate constraints on the parameters of this class of gravity theories using S2-like star orbits under uncertainty of 10 mas. In Section §2 the type of used gravitational potential is given. In Section §3 we present the S2-like stars orbits, gravity parameters and angles of orbital precession, and also compared theoretical results with observations. The main conclusions are pointed out in §4.

## II. THEORY

$R^n$  gravity belongs to power-law fourth-order theories of gravity obtained by replacing the scalar curvature  $R$  with  $f(R) = f_0 R^n$  in the gravity Lagrangian [1, 2]. As a result, in the weak field limit [13], the gravitational potential is found to be [1, 2]:

$$\Phi(r) = -\frac{GM}{2r} \left[ 1 + \left( \frac{r}{r_c} \right)^\beta \right], \quad (1)$$

where  $r_c$  is an arbitrary parameter, depending on the typical scale of the considered system and  $\beta$  is a universal parameter:

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \quad (2)$$

This formula corresponds to a modification of the gravity action in the form:

$$A = \int d^4x \sqrt{-g} (f(R) + L_m), \quad (3)$$

where  $f(R)$  is a generic function of the Ricci scalar curvature and  $L_m$  is the standard matter Lagrangian.

For  $n = 1$  and  $\beta = 0$  the  $R^n$  potential reduces to the Newtonian one, as expected. Parameter  $\beta$  controls the shape of the correction term and is related to  $n$  which is part of the gravity Lagrangian. Since it is the same for all gravitating systems, as a consequence,  $\beta$  must be the

same for all of them and therefore it is universal parameter [2]. The parameter  $r_c$  is the scalelength parameter and is related to the boundary conditions and the mass of the system [2].

## III. RESULTS

### A. Orbits of S2-like stars and parameters of $R^n$ gravity

In order to study the effects of  $R^n$  gravity on motion of the star S2, we performed two-body calculations of its orbit in the  $R^n$  potential (Eq.(1)) during two periods. We assumed the following input parameters taken from the paper of Zakharov et al. [10]: orbital eccentricity of S2-like star  $e = 0.87$ , major semi-axis  $a = 919$  AU, mass of S2-like star  $M_\star = 1 M_\odot$ , mass of central black hole  $M_{BH} = 3.4 \times 10^6 M_\odot$  (where  $M_\odot$  is solar mass) and orbital period of S2-like star is 15 years. We calculated S2-like star orbit during two periods using Newtonian and  $R^n$  potentials. We also investigated the constraints on the parameters  $\beta$  and  $r_c$  for which the deviations between the S2-like stars orbits in the  $R^n$  gravity potential (Eq.(1)) and its Keplerian orbit will stay within the maximum precision of the current instruments (about 10 mas), during one orbital period.

In Fig. 1 we presented trajectory of S2-like star around massive black hole in  $R^n$  gravity (blue solid line) and in Newtonian gravity (red dashed line) for  $r_c = 100$  AU and for the following nine values of parameter  $\beta$ : 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.0475. The black hole is assumed to be located at coordinate origin. We fixed a value of parameter  $r_c$  on 100 AU because this value corresponds to maximal value of parameter  $\beta$  in the parameter space (see Fig. 3) and varied values of parameter  $\beta$ . All 9 presented orbits fulfill request that  $R^n$  orbit and corresponding Newtonian orbit differ less than 10 mas, (i.e. within the maximum precision of the current observations) during one orbital period. We can see that if parameter  $\beta$  increases  $R^n$  orbit differs more from the corresponding Newtonian orbit since the precession angle becomes larger. This indicates that the value of  $\beta$  should be small, as inferred from Solar system data [9], and in contrast to the value  $\beta = 0.817$  (obtained by [2] which gives excellent agreement between theoretical and observed rotation curves). In the future, with improvements in observational facilities the precision on constraints on values of parameters  $\beta$  and  $r_c$  will increase, as well as the accuracy of the S2 orbit.

The corresponding distances between the S2-like star and black hole as a function of time for the same values of parameters  $r_c$  and  $\beta$  as in the Fig. 1 are presented in Fig. 2. There is an additional requirement on parameter space: period of S2-like star orbit has to remain  $\approx 15 \pm 0.2$  yr. Like in previous case, with increasing observational accuracy of period the precision on constraints on values of parameters  $\beta$  and  $r_c$  will also increase.

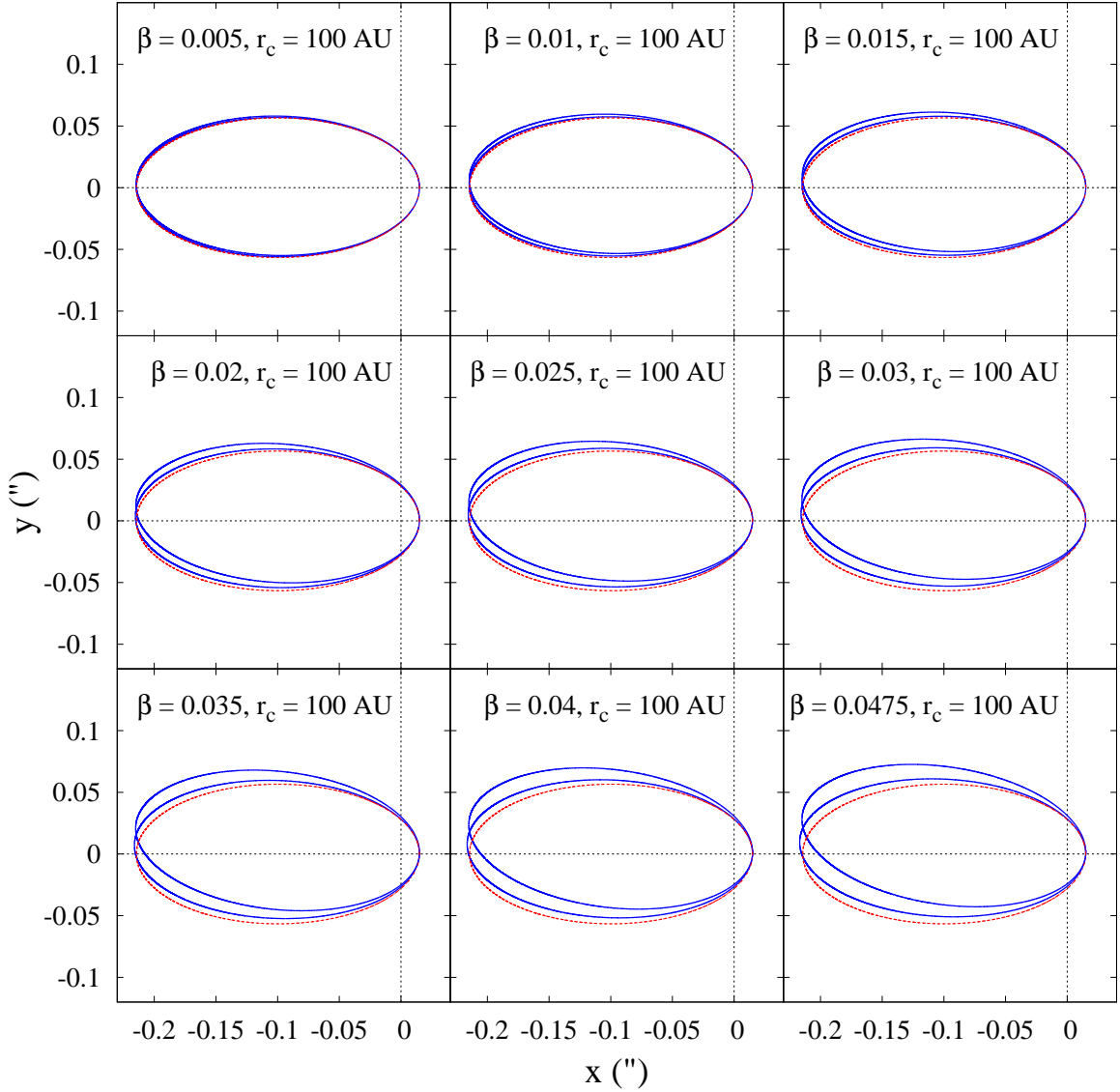


FIG. 1: The orbits of an S2-like star around massive black hole in  $R^n$  gravity (blue solid line) and in Newtonian gravity (red dashed line) for  $r_c = 100$  AU and for these nine values of parameter  $\beta$ : 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.0475. The black hole with mass  $M_{BH} = 3.4 \times 10^6 M_\odot$  is assumed to be located at coordinate origin, and mass of S2-like star is taken to be  $M_\star = 1 M_\odot$ .

In Fig. 3 we presented the parameter space for  $R^n$  gravity under constrain that, during one orbital period, S2-like star orbits under  $R^n$  gravity differ less than  $\varepsilon$  from their orbits under Newtonian gravity for 10 values of parameter  $\varepsilon$ : 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009 and  $0''.01$ . For  $\varepsilon = 0''.01$ , we can see that the maximal value of  $\beta$  is 0.0475, and the corresponding  $r_c$  is 100 AU. That is why we investigate combinations  $\beta \leq 0.0475$  and  $r_c = 100$  AU. This study is important because with improvements in observational facilities the precision of  $\varepsilon$  will increase.

In Fig. 4 we presented the parameter space for  $R^n$  gravity under constraint that, during one orbital period, S2-like star orbits in  $R^n$  gravity differ less than  $\varepsilon$  from the corresponding orbits in Newtonian gravity for 12 values

of parameter  $\varepsilon$ : 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.11 and  $0''.12$ . Although these values are unrealistic, since they are larger than the current observational resolutions, they can be used to obtain tendency of  $\beta$  and  $r_c$  dependences on  $\varepsilon$ .

We want to find how the constraints on values of parameters  $\beta$  and  $r_c$  will change with increasing of  $\varepsilon$ . From Figs. 3 and 4 we can see that smaller values of  $\varepsilon$  will significantly reduce parameter space of  $R^n$  gravity within the given precision.

In Fig. 5(left and right) we presented dependence of the maximal value of parameter  $\beta$  versus  $\varepsilon$  for  $0 \leq \varepsilon \leq 0''.3$  and  $0 \leq \varepsilon \leq 0''.1$ , respectively. We can see that in the region of interest the dependence of the maximal value of parameter  $\beta$  versus values of  $\varepsilon$  is almost strictly

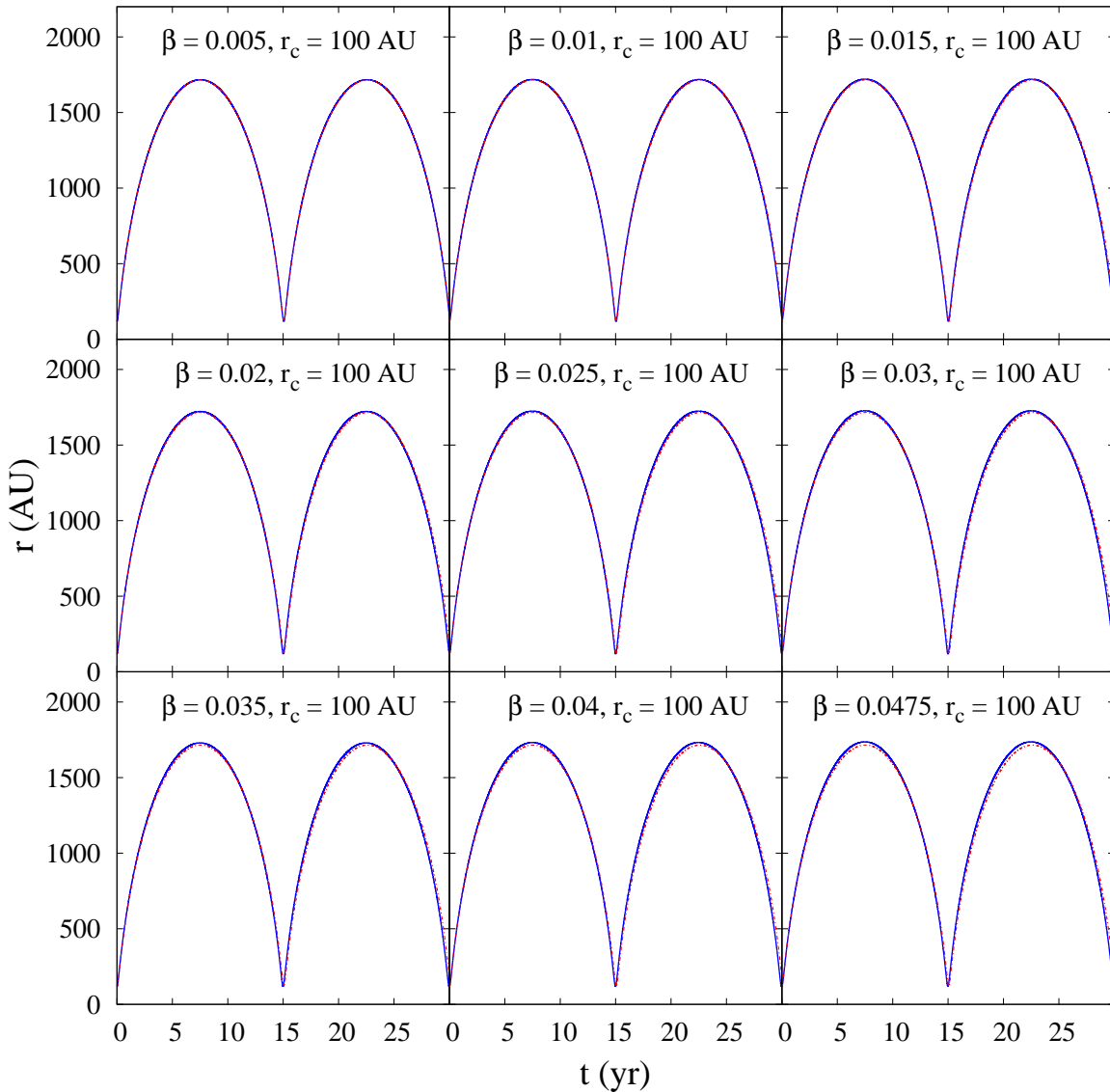


FIG. 2: The distances between the S2-like star and black hole as a function of time for the same values of parameters  $r_c$  and  $\beta$  as in the Fig. 1.

linear.

In Fig. 6(left and right) we presented dependence of the value of parameter  $r_c^{max}$ , that correspond to maximal value of parameter  $\beta$  (e.g.  $\beta^{max}$ ), versus  $\varepsilon$  for  $0 \leq \varepsilon \leq 0''.3$  and  $0 \leq \varepsilon \leq 0''.1$ , respectively. With decrease of  $\varepsilon$  the value of parameter  $r_c^{max}$  increase and in the region of interest the value of  $r_c^{max}$  is near 100 AU.

The trajectories of S2-like star around massive black hole in  $R^n$  gravity (blue solid line) and in Newtonian gravity (red dashed line) are presented in Fig. 7 (left and right) for  $r_c = 100$  AU and  $\beta = 0.02$  during 0.8 and 10 periods, respectively. We can see that the precession of S2-like star orbit is in the clockwise direction in the case when the revolution of S2-like star is in counter clockwise direction.

In Figure 8 we show calculated S2-like star orbits for 11

periods, assuming  $r_c = 100$  AU and these nine values of parameter  $\beta$ : 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.0475. We adopted value  $r_c = 100$  AU because it corresponds to the largest allowed range of parameter  $\beta$ . We obtained that the  $R^n$  gravity causes periastron shifts which result in rosette shaped orbits.

### B. Angle of orbital precession in $R^n$ gravity

Talmadge et al. [14] used post-Newtonian formalism of metric theories of gravity in order to calculate perihelion precession in a potential which deviates from Newtonian potential only slightly. Adkins and McDonnell [15] calculated the precession of Keplerian orbits under the influence of arbitrary central force perturbations. For some

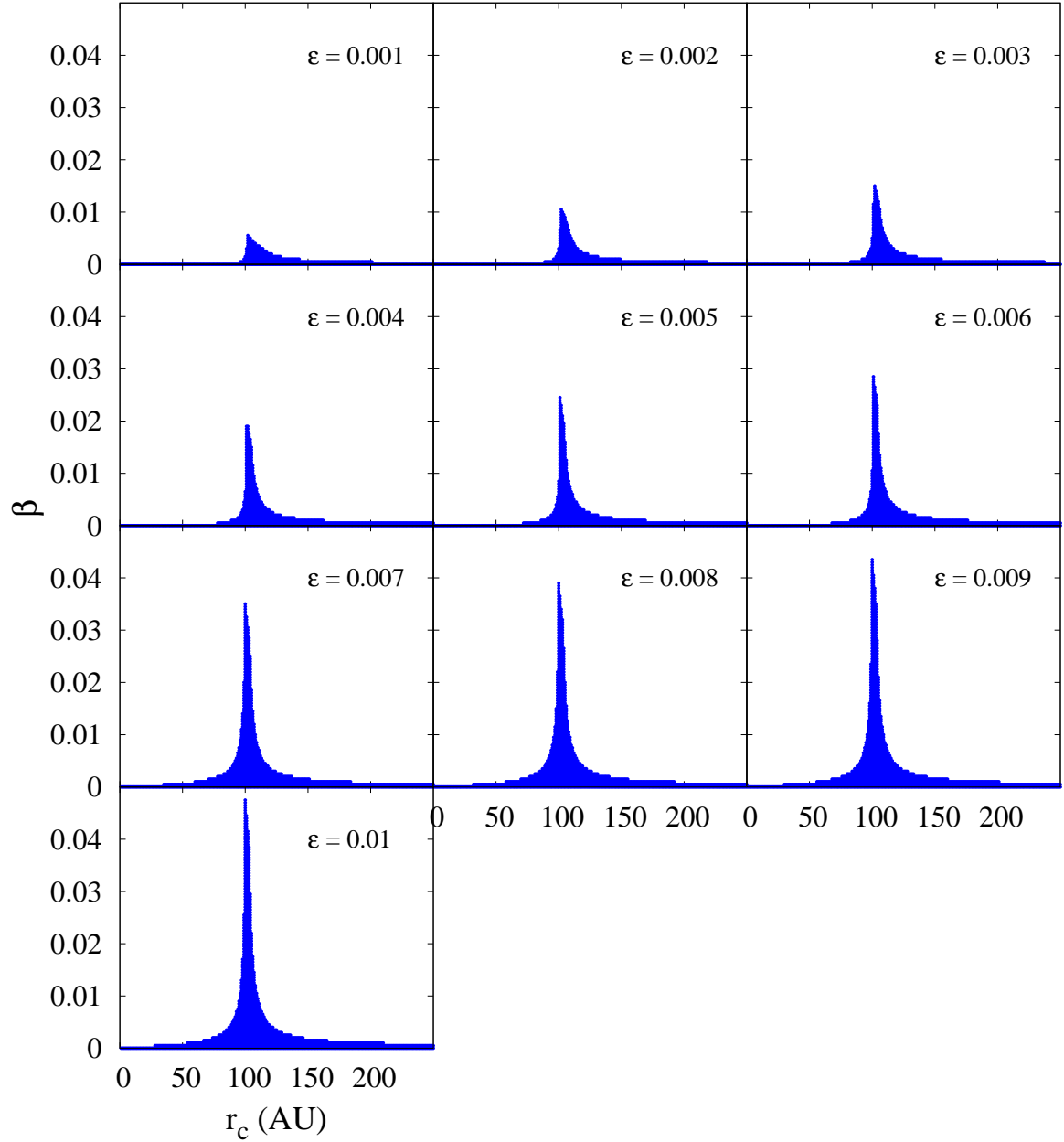


FIG. 3: The parameter space for  $R^n$  gravity under the constraint that, during one orbital period, S2-like star orbits in  $R^n$  gravity differ less than  $\varepsilon$  from the corresponding orbits in Newtonian gravity, for the following 10 values of parameter  $\varepsilon$ : 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009 and 0.01.

examples including the Yukawa potential they presented the results using hypergeometric functions. Schmidt [16] calculated the perihelion precession of nearly circular orbits in a central potential which has a form of modified Newtonian potential. Since the S2-like star orbit is very eccentric, and hence non-circular, we used approach from [15] to obtain analytical expression for precession angle in  $R^n$  gravity.

Assuming a potential which does not differ significantly from Newtonian potential, in our case  $R^n$  gravitational potential, we will derive formula for precession angle of the modified orbit, during one orbital period. First step

is to derive perturbing potential from:

$$V(r) = \Phi(r) - \Phi_N(r) ; \quad \Phi_N(r) = -\frac{GM}{r} . \quad (4)$$

Obtained perturbing potential is of the form:

$$V(r) = -\frac{GM}{2r} \left( \left( \frac{r}{r_c} \right)^\beta - 1 \right) , \quad (5)$$

and it can be used for calculating the precession angle according to the equation (30) from paper [15]:

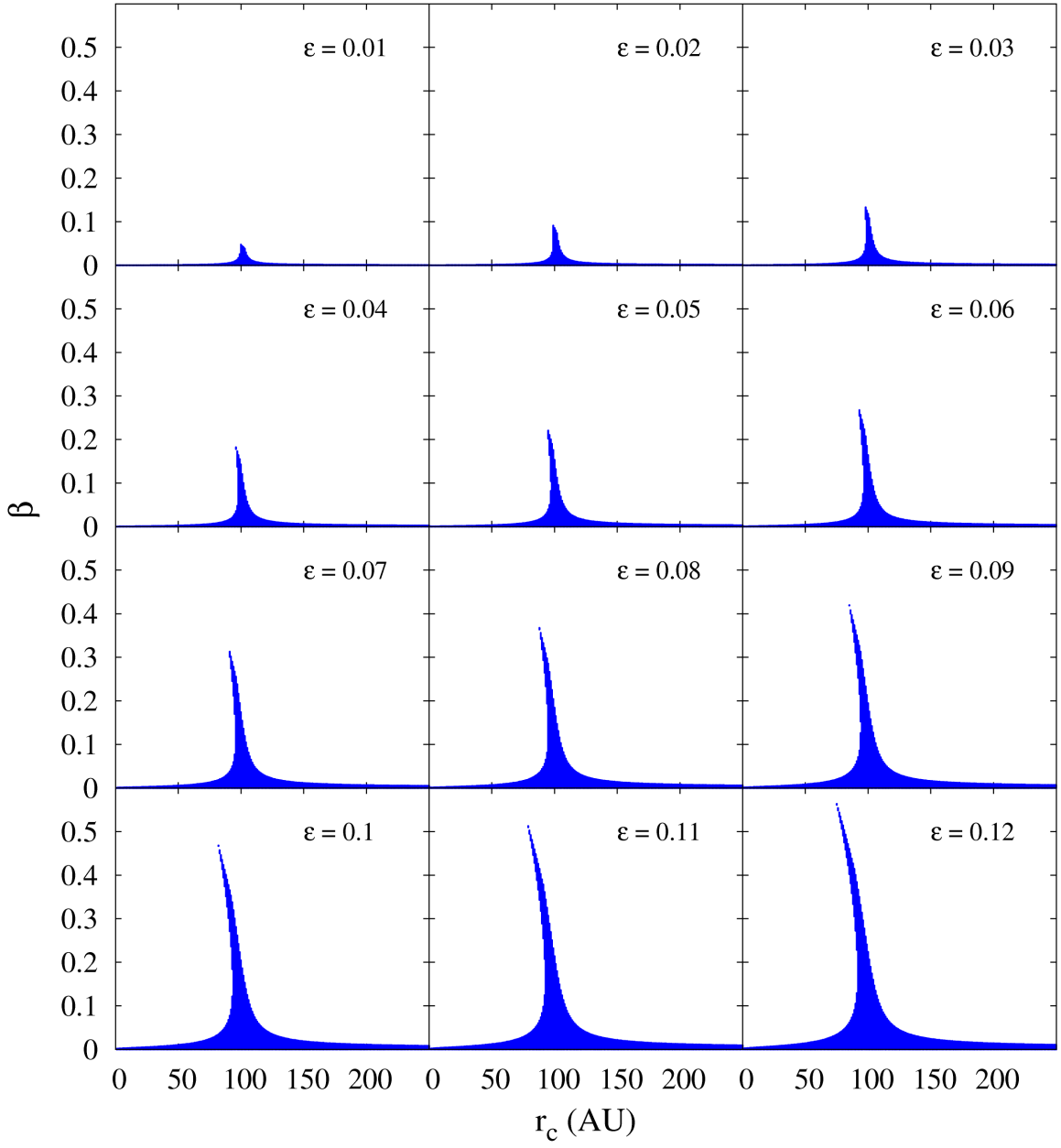


FIG. 4: The same as in Fig. 3 but for the following 12 values of parameter  $\varepsilon$ : 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.11 and 0.12.

$$\Delta\theta = \frac{-2L}{GMe^2} \int_{-1}^1 \frac{z \cdot dz}{\sqrt{1-z^2}} \frac{dV(z)}{dz}, \quad (6)$$

where  $r$  is related to  $z$  via:  $r = \frac{L}{1+ez}$ . By differentiating the perturbing potential  $V(z)$  and substituting its derivative and expression for the semilatus rectum of the orbital ellipse ( $L = a(1-e^2)$ ) in above equation (6), we obtain:

$$\Delta\theta = \frac{\pi}{2} \beta(\beta-1) \left( \frac{a(1-e^2)}{r_c} \right)^\beta \times {}_2F_1 \left( \frac{\beta+1}{2}, \frac{\beta+2}{2}; 2; e^2 \right), \quad (7)$$

where  ${}_2F_1$  is hypergeometric function. The graphical presentation of the precession angle  $\Delta\theta$  for S2-like star orbit as a function of  $\beta$  is given in Fig. 9 (black solid line). From this figure it can be seen that  $\Delta\theta$  is negative for all values of  $\beta$  between 0 and 1, which are of interest in the case of S2-like star orbit.

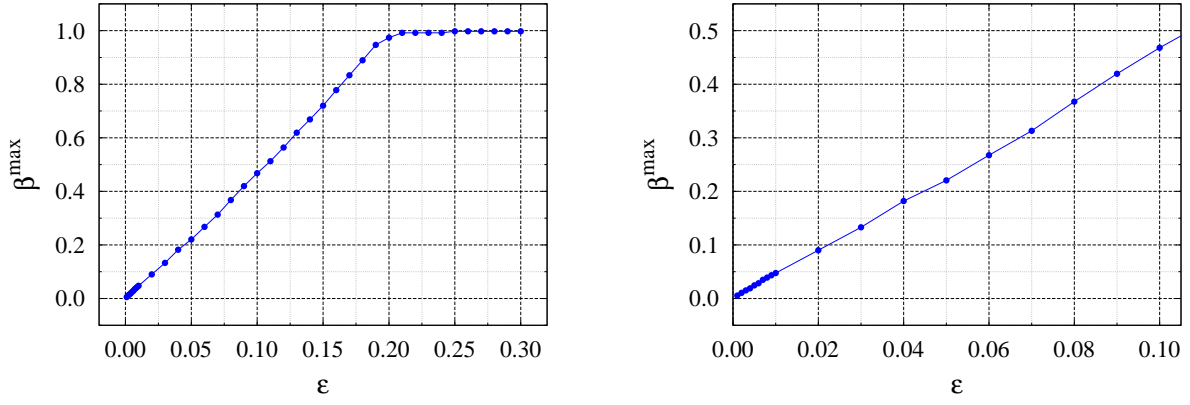


FIG. 5: The dependence of the maximal value of parameter  $\beta$  on precision  $\varepsilon$  ranging from 0 to  $0''.3$  (left) and from 0 to  $0''.1$  (right).

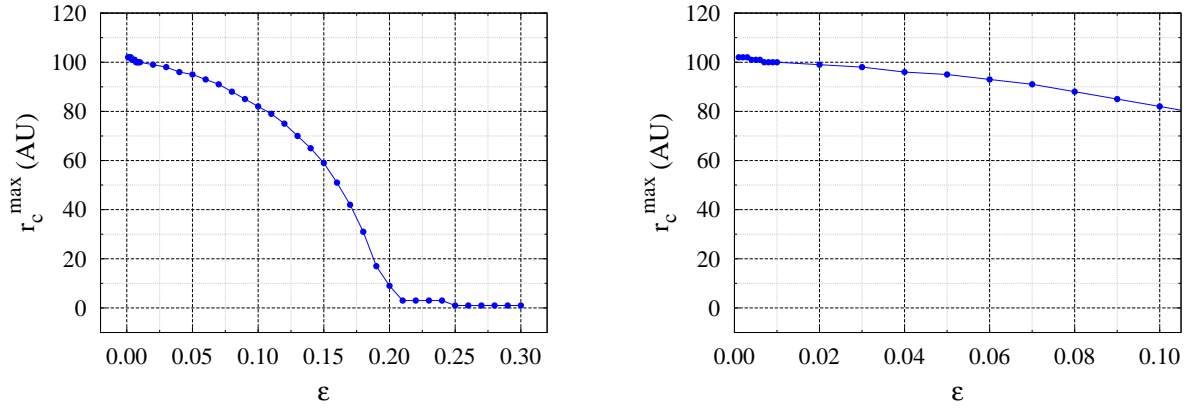


FIG. 6: The dependence of the  $r_c^{max}$  on precision  $\varepsilon$  ranging from 0 to  $0''.3$  (left) and from 0 to  $0''.1$  (right).

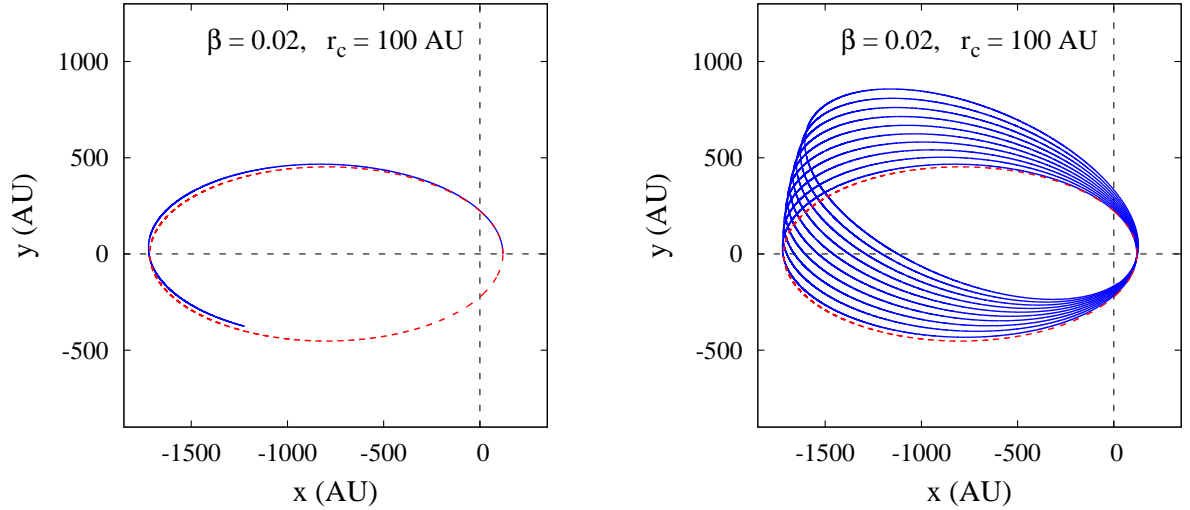


FIG. 7: The orbits of S2-like star around massive black hole in  $R^n$  gravity (blue solid line) and in Newtonian gravity (red dashed line) for  $r_c = 100$  AU and  $\beta = 0.02$  during 0.8 periods (left) and 10 periods (right).

Exact expression (7) is inappropriate for practical applications. However, it can be easily approximated for  $\beta \approx 0$  and  $\beta \approx 1$ . In case of  $\beta \approx 0$  expansion of Eq. (7)

in Taylor's series over  $\beta$ , up to the first order, leads to the following expression for precession angle:

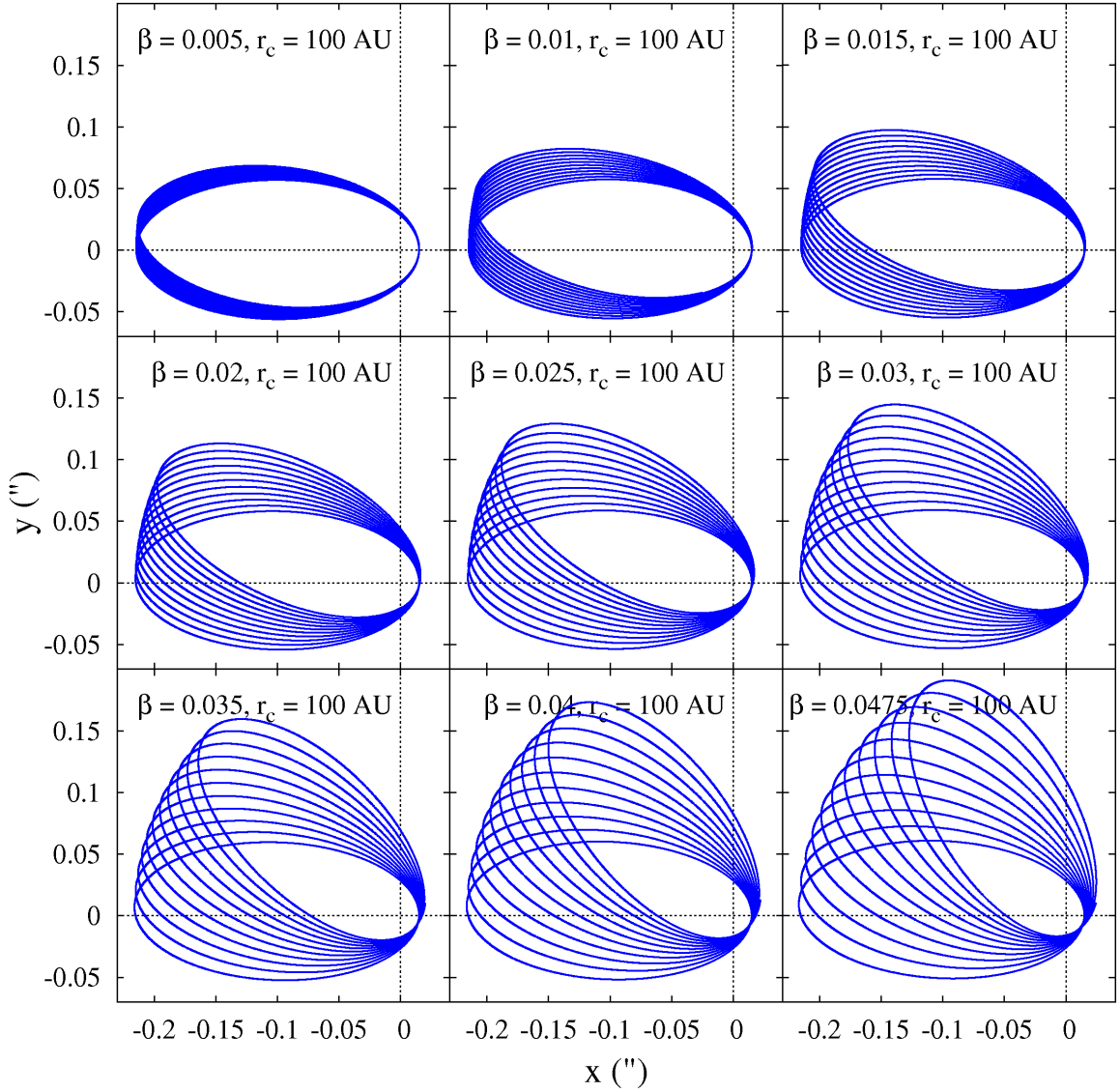


FIG. 8: The orbital precession of an S2-like star around massive black hole located at coordinate origin in  $R^n$  gravity for  $r_c = 100$  AU and these nine values of parameter  $\beta$ : 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.0475.

$$\begin{aligned}
 \Delta\theta &= \frac{\pi^{rad}\beta(\sqrt{1-e^2}-1)}{e^2} \\
 &= \frac{180^\circ\beta(\sqrt{1-e^2}-1)}{e^2}.
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \Delta\theta &= \frac{\pi^{rad}a(\beta-1)(\sqrt{1-e^2}-1+e^2)}{r_ce^2} \\
 &= \frac{180^\circ a(\beta-1)(\sqrt{1-e^2}-1+e^2)}{r_ce^2}.
 \end{aligned} \tag{9}$$

Above expression in the case of S2-like star orbit is presented in Fig. 9 as a blue dash-dotted line. Similarly, expansion of Eq. (7) in power series for  $\beta \approx 1$ , leads to the following expression for precession angle (red dotted line in Fig. 9):

One can expect that in general precession angle depends on semimajor axis and eccentricity of the orbit (see e.g. Iorio & Ruggiero 2008 [17]), as well as on both potential parameters  $\beta$  and  $r_c$ . It is indeed case for  $\beta \approx 1$  in Eq. (9). But as it can be seen from formula (8), the precession angle in the case when  $\beta$  is small ( $\beta \approx 0$ ) depends only on eccentricity and universal constant  $\beta$  itself.

In order to test if the approximation from Eq. (8) is



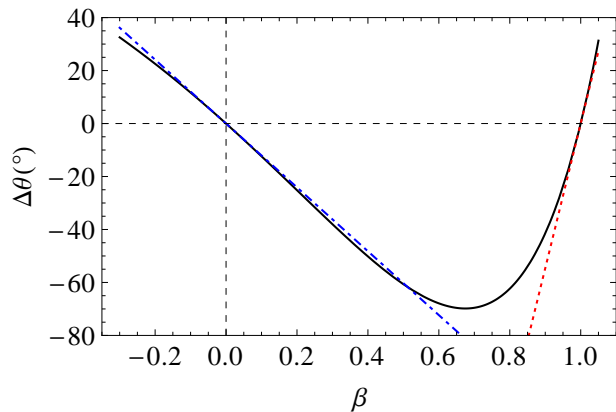


FIG. 9: The exact expression for precession angle  $\Delta\theta$  in  $R^n$  gravity (in degrees) as a function of the parameter  $\beta$  (black solid line) and two its approximations: for  $\beta \approx 0$  (blue dash-dotted line) and for  $\beta \approx 1$  (red dotted line). The other parameters correspond to the case of S2-like star:  $a = 919$  AU,  $e = 0.87$  and  $r_c = 100$  AU.

satisfactory in case of S2-like star, we derived its precession angle in two ways:

- analytically from the approximative formula (8)
- numerically from calculated orbits presented in Fig. 8.

Comparison of the obtained precession angles by these two methods is presented in Table I. As it can be seen from this table, the approximative formula (8) can be used for estimating the precession angle for all values of  $\beta$  from Fig. 8.

Above analysis indicates that  $R^n$  gravity results with retrograde shift of S2-like star orbit. Rubilar and Eckart [11] showed that the orbital precession can be due to relativistic effects, resulting in a prograde shift, or due to a extended mass distribution, producing a retrograde shift. We can conclude that perturbing potential  $V(r)$  has a similar effect as extended mass distribution, since it produces a retrograde orbital shift.

Since the precession has negative direction, as in the case of extended mass distribution, the obtained results are useful for testing if the precession due to extended dark matter enclosed into orbit of S2-like star could be also explained by  $R^n$  gravity. If this is possible, it will exclude the need for dark matter hypothesis. Therefore, if future and more precise observations of bright stars near the Galactic Center will show a precession in the negative direction, we have to conclude that the phenomenon could be caused by bulk distributions of stellar cluster or/and dark matter in classical Newtonian (GR) gravity or by  $R^n$  gravity. On the other hand, if there is no deviations from Newtonian (GR) trajectories with an accuracy of observations, one could put constraints on stellar cluster and dark matter distributions and on parameters of  $R^n$  gravity if we adopt the theory to fit observational data.

TABLE I: The numerically calculated by computer simulation ( $\Delta\theta^o$ ) and analytically calculated from Eq. (8) ( $\Delta\theta^c$ ) values of precession angle of S2-like star orbit (in degrees) as a function of universal constant  $\beta$  of  $R^n$  gravity with parameter  $r_c = 100$  AU.

$\beta$	$\Delta\theta^o$	$\Delta\theta^c$
0.005	-0.602	-0.604
0.01	-1.203	-1.209
0.015	-1.802	-1.816
0.02	-2.400	-2.425
0.025	-2.997	-3.035
0.03	-3.592	-3.647
0.035	-4.186	-4.261
0.040	-4.779	-4.876
0.045	-5.666	-5.493

### C. Comparison between the theoretical results and observations

Here we compare the obtained theoretical results for S2-like star orbits in the  $R^n$  potential with two independent sets of observations of the S2 star, obtained by New Technology Telescope/Very Large Telescope (NTT/VLT), as well as by Keck telescope [see Fig. 1 in 4], which are publicly available as the supplementary on-line data to the electronic version of the paper [4]. However, all the above two-body simulations in  $R^n$  gravity potential resulted with the true orbits of S2-like stars, i.e. the simulated positions of S2-like stars presented in Figs. 1, 2, 7 and 8 are in their orbital planes. Therefore, in order to compare them with observed positions, the first step is to project them to the observer's sky plane, i.e. to calculate the corresponding apparent orbits. From the theory of binary stars it is well known that any point  $(x, y)$  on the true orbit could be projected into the point  $(x^c, y^c)$  on the apparent orbit according to [see e.g. 19, 20]:

$$x^c = l_1 x + l_2 y, \quad y^c = m_1 x + m_2 y, \quad (10)$$

where the expressions for  $l_1, l_2, m_1$  and  $m_2$  depend on three orbital elements ( $\Omega$  - longitude of the ascending node,  $\omega$  - longitude of pericenter and  $i$  - inclination) [19, 20]. One should take into account that in the case of orbital precession  $\omega$  is a function of time, and therefore should be in general treated as an adjustable parameter during the fitting procedure. However, the previously mentioned theoretical and observational results showed that in the case of S2-like stars this precession is most likely very small, and hence we assumed  $\omega$  as a constant when projecting true positions to their corresponding apparent values. For that purpose we used the fol-

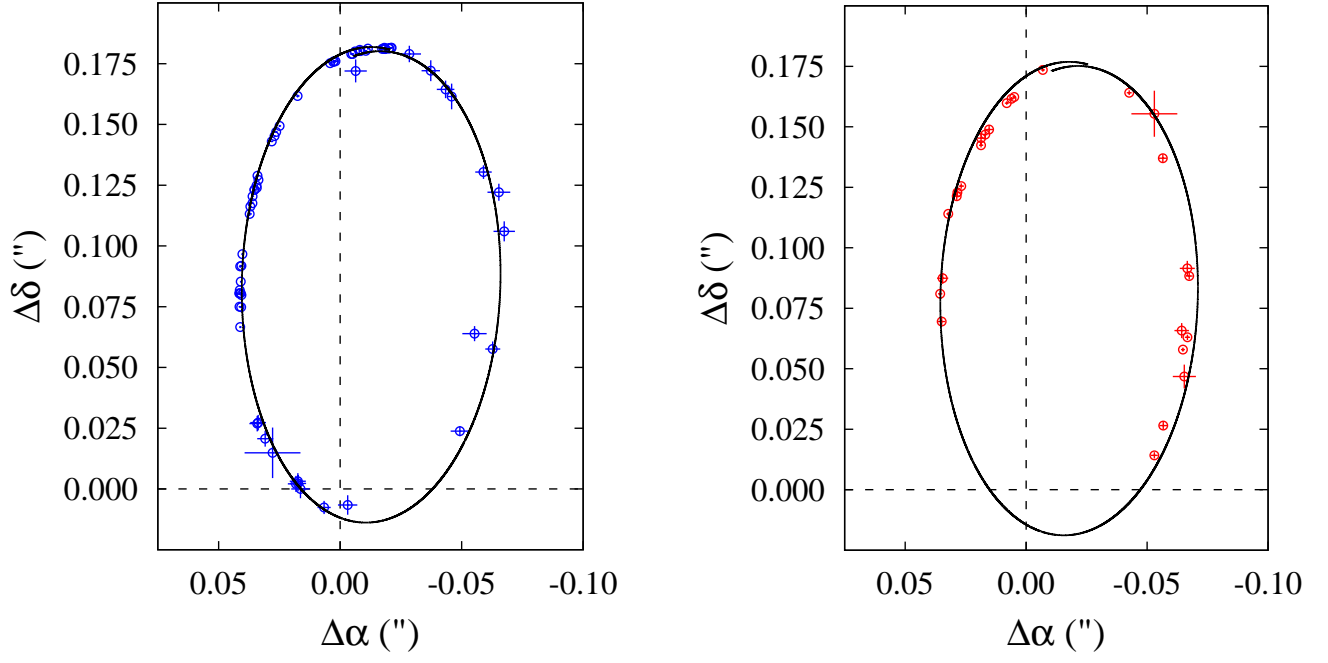


FIG. 10: The fitted orbit of S2 star around massive black hole in  $R^n$  gravity for  $r_c = 100$  AU and  $\beta = 0.01$  (black solid lines in both panels). The NTT/VLT astrometric observations are presented in the left panel by blue circles, while the Keck measurements are denoted by red circles in the right panel.

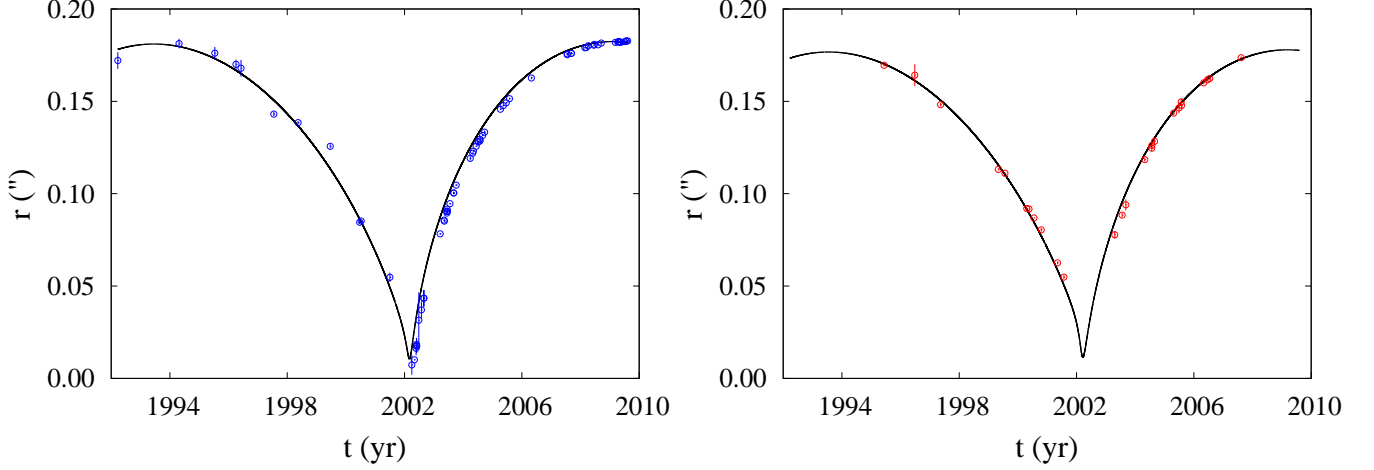


FIG. 11: Comparison between the fitted (black solid lines) and measured (open circles) distances of the S2 star from black hole in the case of NTT/VLT (left) and Keck (right) observations.

lowing Keplerian orbital elements from [4]:  $i = 134^\circ.87$ ,  $\Omega = 226^\circ.53$  and  $\omega = 64^\circ.98$ . Besides, our previous theoretical results indicated that the most likely value of the scale parameter is  $r_c \approx 100$  AU, and we adopted that value in order to reduce the number of free parameters when fitting the observations.

We fitted the observed orbits of S2 star using the following procedure:

1. initial values for S2 star true position  $(x_0, y_0)$ , orbital velocity  $(\dot{x}_0, \dot{y}_0)$  and the parameter  $\beta$  of  $R^n$  gravity potential are specified;
2. the positions  $(x_i, y_i)$  of the S2 star along its true orbit are calculated at the observed epochs using two-body simulations in the  $R^n$  gravity potential, assuming that distance to the S2 star is  $d_\star = 8.3$  kpc and mass of central black hole  $M_{BH} = 4.4 \times 10^6 M_\odot$  [4];
3. the corresponding positions  $(x_i^c, y_i^c)$  along the apparent orbit are calculated using the expression (10);
4. the root mean square  $(O - C)$  goodness of fit is es-

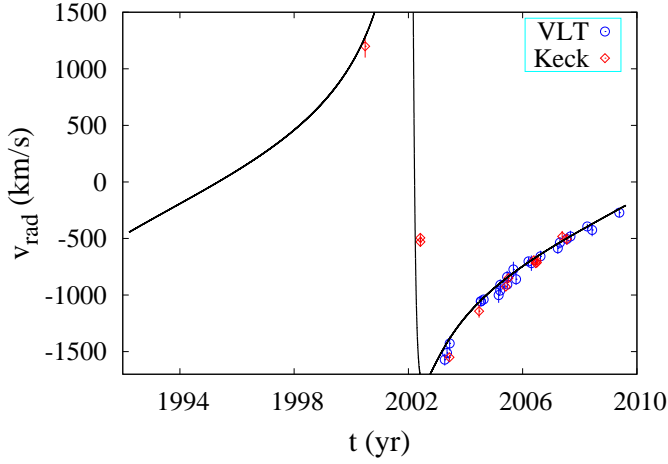


FIG. 12: Comparison between the fitted (black solid line) and measured radial velocities for the S2 star. Measured velocities are labeled with open circles (VLT data) and open rhombuses (Keck data).

timated according the following expression:

$$(O - C)_{rms} = \sqrt{\frac{\sum_{i=1}^N [(x_i^o - x_i^c)^2 + (y_i^o - y_i^c)^2]}{2N}},$$

where  $(x_i^o, y_i^o)$  is the  $i$ -th observed position,  $(x_i^c, y_i^c)$  is the corresponding calculated position, and  $N$  is the number of observations;

- the values of the input parameters are varied and the procedure is repeated until the minimum of  $(O - C)_{rms}$  is reached.

The best fit is obtained for the following small value of the universal constant:  $\beta = 0.01$ , in which case the corresponding precession is around  $-1^\circ$  (see Table I). In Fig. 10 we present two comparisons between the obtained best fit orbit for  $\beta = 0.01$  in the  $R^n$  gravity potential and the positions of S2 star observed by NTT/VLT (left) and Keck (right). The corresponding calculated distances of S2 star from massive black hole are shown in Fig. 11. Astrometric data for the S2 star orbit are presented by blue dots (NTT/VLT measurements) and by red dots (Keck measurements). As one can see from these figures, there is a good agreement between the theoretical orbit and NTT/VLT observations. In case of Keck measurements, we had to move the origin of the coordinate system with respect to the both axes for 5 mas, in order to get reasonable fit. We made this correction following the suggestion from [4], where it was necessary in order to combine the two data sets. After that we also achieved the satisfying agreement between the same fitted orbit and the both NTT/VLT and Keck data sets, in spite the fact that both groups obtained slightly different orbital elements, distance to the S2 star, as well as mass of central black hole [4, 6, 18].

In order to obtain the orbital elements of S2 star both, NTT/VLT and Keck groups, fitted their observations

with Keplerian orbits, but at the same time, they had to allow that the position of the center of mass varies as a function of time, i.e. they had to introduce the orbital precession. As it can be seen from Fig. 10, the orbit of S2 is not closed in vicinity of its apocenter, which clearly shows that the orbital precession is a natural consequence of  $R^n$  gravity. Moreover, by comparing the arcs of orbit near the apocenter with the corresponding results presented in Fig. 1 from [4], one can see that their curvatures are different, which indicates the opposite directions of precession in these two cases. Therefore, the future more precise observations of S2 star positions near its apocenter could have a decisive role in verifying or disproving the validity of  $R^n$  gravity near the Galactic Center.

We also made a comparison between the fitted and measured radial velocities for the S2 star (see Fig. 12). The well known expression for radial velocity in polar coordinates  $r$  and  $\theta$  is [see e.g. 19]:

$$v_{rad} = \sin i \left[ \sin(\theta + \omega) \cdot \dot{r} + r \cos(\theta + \omega) \cdot \dot{\theta} \right]. \quad (11)$$

However, we used the corresponding expression in rectangular coordinates  $x = r \cos \theta$  and  $y = r \sin \theta$  to calculate the fitted radial velocities:

$$v_{rad} = \frac{\sin i}{\sqrt{x^2 + y^2}} [\sin(\theta + \omega) \cdot (x\dot{x} + y\dot{y}) + \cos(\theta + \omega) \cdot (x\dot{y} - y\dot{x})], \quad (12)$$

where  $\theta = \arctan \frac{y}{x}$ . As it can be seen from Fig. 12, the agreement between our theoretical predictions and the observations is also satisfactory.

Although the both observational sets indicate that the orbit of S2 star most likely is not a Keplerian one, the nowadays astrometric limit of around 10 mas is not sufficient to unambiguously confirm such claim. We hope that in the future, it will be possible to measure the stellar positions with astrometric errors several times smaller than errors of current observations.

#### IV. CONCLUSIONS

In this paper S2-like star orbit has been investigated in the framework of fourth order gravity theory. Using the observed positions of S2 star we put new constraints on the parameters of this class of gravity theories. We confirmed that these parameters must be very close to those corresponding to the Newtonian limit of the theory. For parameter  $\beta$  approaching to zero, we recover the value of the Keplerian orbit for S2 star. Also, we performed two-body calculations of its orbit in the  $R^n$  potential. The obtained results showed that, in contrast to General Relativity,  $R^n$  gravity gives retrograde direction of the precession of the S2 orbit, like in the case when it is caused by an extended matter concentration in Newtonian potential.

Despite the excellent agreement between theoretical and observed rotation curves obtained by Capozziello and coworkers [2] for  $R^n$  parameter  $\beta$  (the slope  $n$  of the gravity Lagrangian is set to the value  $n = 3.5$  giving  $\beta = 0.817$ ), our findings indicate that for  $\varepsilon = 0''.01$  maximal value of  $\beta$  is 0.0475, i.e.  $\beta$  is less or equal than 0.0475, and our fitting indicated that optimal value for  $\beta$  is around 0.01. Therefore,  $R^n$  gravity in this form may not represent a good candidate to solve both the dark energy problem on cosmological scales and the dark matter one on galactic scales using the same value of parameter  $\beta$ . But this theory has its own benefits in explaining orbits of the stars and solar system data.

For today astrometric limit of around 10 mas for S2 star orbit, within that limit one can not say for sure that S2 star orbit really deviates from the Newtonian case, i.e. we have to stress that at the moment observations are in agreement with the Newtonian point-like potential for the Galactic Center. Therefore the observations and their theoretical analysis give us one of the best cases to discuss departures from the standard GR plus stellar cluster and dark matter (as it was done in our papers and papers of other authors) or to analyze an opportunity to get

constraints on alternative theories observing trajectories of S2-like stars. The newest astrometric data for the star S2 of NTT/VLT measurements and Keck measurements show the Keplerian orbit fits for the respective data set, do not yield closed ellipses. Maybe this represents really small deviation of S2 star orbit from the Newtonian case and for more sure conclusion we need astrometric errors several times smaller than these errors, but we compared these data with S2 star orbit obtained using  $R^n$  gravity potential.

We can conclude that additional term in  $R^n$  gravity compared to Newtonian gravity has a similar effect like extended mass distribution and produce a retrograde shift, that results in rosette shaped orbits.

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